

Cross Country Evidence for the EPK Puzzle

Maria Grith

Wolfgang Karl Härdle

Andrija Mihoci

Ladislav von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
and Economics

Humboldt-Universität zu Berlin

<http://lvb.wiwi.hu-berlin.de>

<http://www.case.hu-berlin.de>



Motivation

- Pricing kernel (PK)
 - ▶ Consumption based models
 - marginal rate of consumption substitution
 - ▶ Arbitrage free models
 - Radon-Nikodym derivative of the physical measure w.r.t. the risk neutral measure
 - ▶ Risk Neutral Valuation
 - ▶ PK - Black-Scholes
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- Empirical pricing kernel (EPK)
 - ▶ $\hat{\mathcal{K}}$ any estimate of the PK
 - ▶ EPK paradox - locally increasing EPK



PK Estimation

- Indirect estimation of the PK

$$\hat{\kappa} = \frac{\hat{q}}{\hat{p}}$$

- ▶ q risk neutral density; p physical density;
- ▶ European options and stock index data
- ▶ EPK puzzle emerges
Ait-Sahalia and Lo (2000), Brown and Jackwerth (2004)



PK Estimation

- Direct estimation of the PK

$$\hat{\mathcal{K}} = G_{\hat{\theta}}$$

- ▶ $PK \stackrel{\text{def}}{=} G_{\theta} \propto U'$, U aggregated utility
- ▶ cross-sectional equity returns data
- ▶ mixed evidence for the EPK puzzle
Dittmar (2002), Schweri (2011)



EPK Paradox: European option market

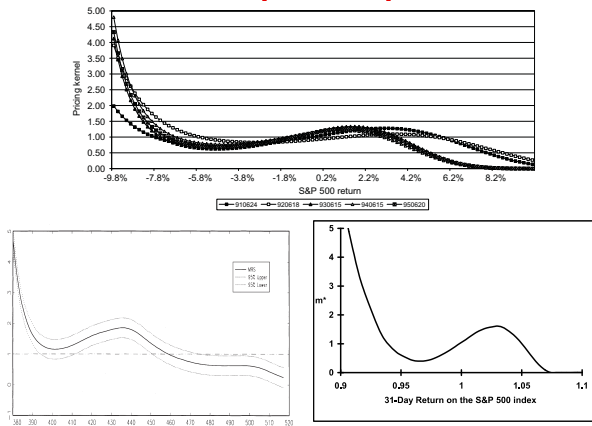


Figure 1: EPK's: Engle and Rosenberg (2002), Ait-Sahalia and Lo (2000), Brown and Jackwerth (2004)

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EPK Paradox: European option market

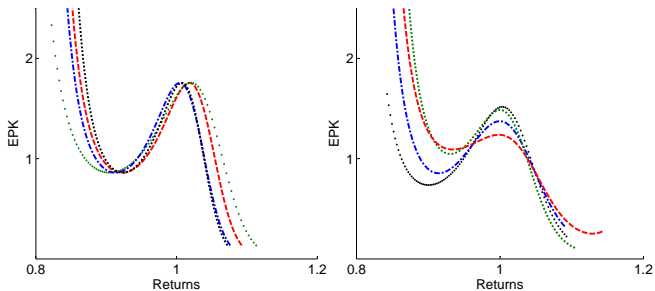


Figure 2: EPK's for various maturities (left) and different estimation dates for fixed maturity 1M (right), Grith et al. (2010)



EPK Paradox: European option market

Figure 3: EPK's across moneyness κ and maturity τ for DAX from 20010101 – 20011231, Giacomini and Härdle (2008)

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EPK Paradox

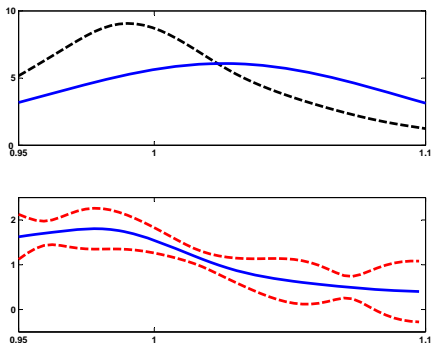


Figure 4: Upper panel: estimated risk neutral density \hat{q} and historical density \hat{p} . Lower panel: EPK and 95% uniform confidence bands on 20060228, Härdle et al. (2010)

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Research Questions

- Parametrization of the PK that admits nonmonotonicity
- Dynamic estimation of the EPK parameters
- Test the significance of the 'bump' in the EPK
- Cross-country variation of the EPK in equity returns



Outline

1. Motivation ✓
2. Pricing Kernel (PK)
3. Generalized Method of Moments (GMM)
4. Empirical Results
5. Conclusion



Modeling Framework

□ Neoclassical economy, representative agent

- ▶ Exogenous income ω_t
- ▶ Consumption c_t and financial portfolio of k assets

$$\omega_t = c_t + q_t^\top S_t$$

Asset holdings $q_t = (q_{1,t}, \dots, q_{k,t})^\top$, prices $S_t = (S_{1,t}, \dots, S_{k,t})^\top$

- ▶ c_{t+1} contains the future income and all asset payoffs

$$c_{t+1} = \omega_{t+1} + q_t^\top S_{t+1}$$



Preferences

- Expected time separable and state-dependent utility

$$u(c_t, c_{t+1}) = u(c_t) + \beta_1 E_t[u(c_{t+1})] I\{c_t \in [0, x)\} \\ + \beta_2 E_t[u(c_{t+1})] I\{c_t \in [x, \infty)\}$$

- ▶ Reference point x , preference parameters β_1 and β_2
- ▶ $E_t[\bullet] = E[\bullet | \mathcal{F}_t]$



Optimal Portfolio Holding

$$\begin{aligned} \max_{c_t, c_{t+1}} u(c_t, c_{t+1}) = & \max_{q_t} [u(\omega_t - S_t^\top q_t) \\ & + \beta_1 E_t [u(\omega_{t+1} + q_t^\top S_{t+1})] I \{ (\omega_{t+1} + q_t^\top S_{t+1}) \in [0, x] \} \\ & + \beta_2 E_t [u(\omega_{t+1} + q_t^\top S_{t+1})] I \{ (\omega_{t+1} + q_t^\top S_{t+1}) \in [x, \infty) \}] \end{aligned}$$

- Consumption based asset pricing

$$S_t = E_t \left[\left\{ \beta_1 \frac{u'(c_{t+1})}{u'(c_t)} I \{ c_t \in [0, x] \} + \beta_2 \frac{u'(c_{t+1})}{u'(c_t)} I \{ c_t \in [x, \infty) \} \right\} S_{t+1} \right] \quad (1)$$



Preferences

- Power utility $u(x) = x^{1-\gamma}/(1-\gamma)$

$$\frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}.$$

constant relative risk aversion coefficient (CRRA) $\gamma > 0$



Pricing Kernel

Assumption (Cochrane, 1996)

$$c_{t+1} = r_{m,t+1} = S_{m,t+1}/S_{m,t}$$

□ State dependent pricing kernel from (1)

$$\mathcal{K}_\theta(r_{m,t+1}) = \beta_1 r_{m,t+1}^{-\gamma} \mathbf{1}\{r_{m,t+1} \in [0, x)\} + \beta_2 r_{m,t+1}^{-\gamma} \mathbf{1}\{r_{m,t+1} \in [x, \infty)\}$$

with $\theta = (\beta_1, \beta_2, \gamma)^\top$

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Generalized Method of Moments

- Interpret (1) as the expectation of k moment conditions

$$E_t [\mathcal{K}_\theta (r_{m,t+1}) R_{t+1} - 1_k] = 0_k, \quad (2)$$

where $R_{t+1} = (S_{1,t+1}/S_{1,t}, \dots, S_{k,t+1}/S_{k,t})^\top$. Then for

$$g(\theta) = \mathcal{K}_\theta (r_{m,t+1}) R_{t+1} - 1_k, \quad E_t [g(\theta)] = 0_k$$

the sample analogue of (2)

$$g_n(\theta) = n^{-1} \sum_{t=0}^{n-1} \{\mathcal{K}_\theta (r_{m,t+1}) R_{t+1} - 1_k\} \quad (3)$$

over the data sample of size n .



Two-step GMM

- 1st step: weighting matrix I_k

$$\tilde{\theta}_n \stackrel{\text{def}}{=} \arg \min_{\theta} \left\{ \mathbf{g}_n^\top(\theta) \mathbf{g}_n(\theta) \right\}.$$

$$\tilde{W}_n = n^{-1} \sum_{t=0}^{n-1} \mathbf{g}(\tilde{\theta}_n) \mathbf{g}(\tilde{\theta}_n)^\top.$$

- 2nd step: weighting matrix \tilde{W}_n

$$\hat{\theta}_n \stackrel{\text{def}}{=} \arg \min_{\theta} \left\{ \mathbf{g}_n^\top(\theta) \tilde{W}_n^{-1} \mathbf{g}_n(\theta) \right\}$$



Data

- Cross country analysis
 - ▶ Germany and UK, 1998–2007 (daily data)
 - ▶ Overlapping monthly returns
 - ▶ Rolling window (5y)

- Stock markets
 - ▶ Index returns (DAX, FTSE 100)
 - ▶ Returns of the largest 20 constituents of each market
 - ▶ Reference point: zero simple net market return ($x = 1$); 5y average market return



EPK Dynamics

Figure 5: EPK on the German stock market in 2005. Reference point: zero simple net market return.

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Parameter Dynamics

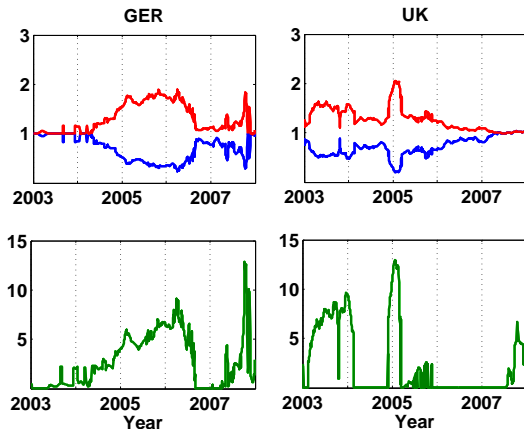


Figure 6: Estimated parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\gamma}$ on the German and the British stock market. Reference point: zero simple net market return.

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Parameter Dynamics

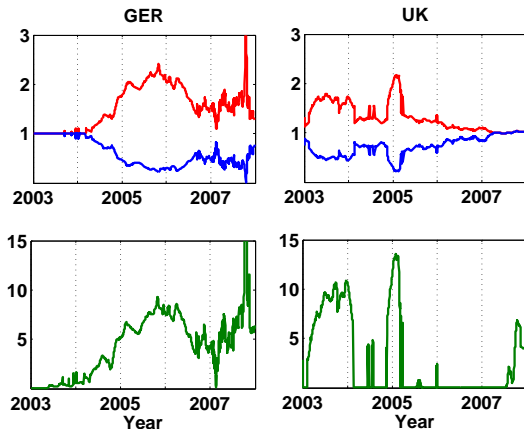


Figure 7: Estimated parameters $\hat{\beta}_1$, $\hat{\beta}_2$ and $\hat{\gamma}$ on the German and the British stock market. Reference point: 5y mean market return.

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EPK Puzzle - Stock Markets

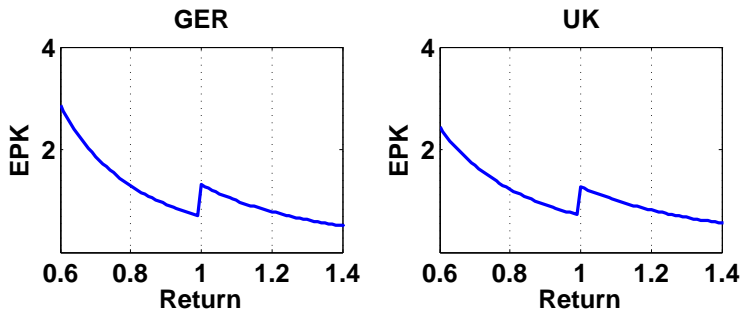


Figure 8: EPK given average estimated parameters from 2003-2007 on the German $\hat{\theta} = (0.69, 1.31, 2.78)^\top$ and the British stock market $\hat{\theta} = (0.72, 1.27, 2.39)^\top$. Reference point: zero simple net market return.



EPK Puzzle - Stock Markets

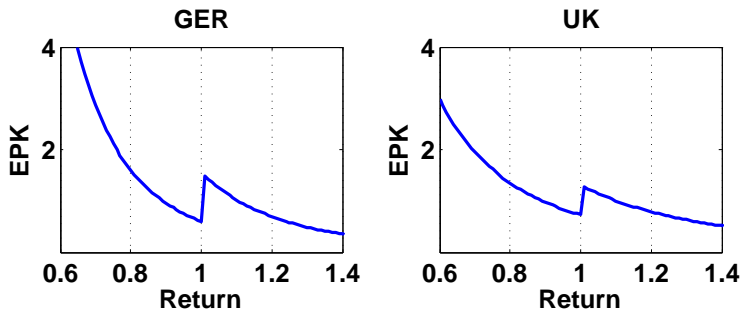


Figure 9: EPK given average estimated parameters from 2003-2007 on the German $\hat{\theta} = (0.60, 1.54, 4.37)^\top$ and the British stock market $\hat{\theta} = (0.73, 1.31, 2.76)^\top$. Reference point: 5y mean market return.



Conclusion

- (i) Estimating pricing kernels (PK)
 - ▶ State-dependent preferences - 'jump' in the PK
 - ▶ Estimated 'jump' is time-persistent with different intensities

- (ii) Cross country study
 - ▶ Evidence for the existence of the 'jump' in both countries
 - ▶ Positive comovements between EPKs' parameters



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Wolfgang Karl Härdle

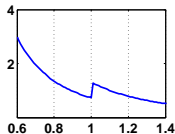
Andrija Mihoci

Ladislav von Bortkiewicz Chair of Statistics
C.A.S.E. – Center for Applied Statistics
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Humboldt-Universität zu Berlin

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Risk Neutral Valuation ► Motivation

- Present value of the payoffs $\psi(S_T)$

$$P_0 = E_Q \left[e^{-Tr} \psi(s_T) \right] = \int_0^\infty e^{-Tr} \psi(s_T) \mathcal{K}(s_T) p(s_T) ds_T$$

r risk free interest rate, $\{S_t\}_{t \in [0, T]}$ stock price process,
 p pdf of S_T , Q risk neutral measure, $\mathcal{K}(\cdot)$ pricing kernel



PK under the Black-Scholes Model ► Motivation

- Geometric Brownian motion for S_t

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

μ mean, σ volatility, W_t Wiener process

- Physical density p is log-normal, $\tau = T - t$

$$p_t(S_T) = \frac{1}{S_T \sqrt{2\pi\sigma^2\tau}} \exp \left[-\frac{1}{2} \left\{ \frac{\log(S_T/S_t) - \left(\mu - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \right\}^2 \right]$$

- Risk neutral density q is log-normal: replace μ by r



PK under the Black-Scholes Model ▶ Motivation

- PK is a decreasing function in S_T for fixed S_t

$$\begin{aligned} \mathcal{K}(S_t, S_T) &= \left(\frac{S_T}{S_t}\right)^{-\frac{\mu-r}{\sigma^2}} \exp\left\{\frac{(\mu-r)(\mu+r-\sigma^2)\tau}{2\sigma^2}\right\} \\ &= b \left(\frac{S_T}{S_t}\right)^{-\delta} \end{aligned}$$

$b = \exp\left\{\frac{(\mu-r)(\mu+r-\sigma^2)\tau}{2\sigma^2}\right\}$ and $\delta = \frac{\mu-r}{\sigma^2} \geq 0$ constant relative risk aversion (CRRA) coefficient

